

# Features of Fast Neutrons in Dark Matter Searches

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## Abstract

Diffraction scattering of “fast” or “high energy ” neutrons can give low energy nuclear recoils in the signal region for dark matter searches. We present a discussion using the ‘black disc’ model. This permits a simple and general, although approximate, description of this possible background. We note a number of its features. In particular there are mass number  $A$  dependent aspects which can be studied in setups where events on different nuclei are observable at the same time. These include the recoil energy distributions, and the  $A$  behavior of the cross section. We define a parameter  $E_R^o$  which characterizes the recoil energy to be expected due to fast neutrons. It ranges from 100 keV on light nuclei to a few keV on heavy nuclei, and a general treatment is possible in terms of it, within the ‘black disc’ approximation. In addition the presence of inelastic processes on the nuclei at about the same level of elastic processes would be characteristic of fast neutrons.

## 1 Introduction

In the search for dark matter by direct detection in the laboratory, spectacular levels of background reduction have been achieved. In the “Commissioning Run” of the CRESST II detector, for example, the rate of possibly interesting events was reported as 0.063 per kg-day [1], based on three events. For the about 600 gm of sensitive material used in the dark matter analysis, this corresponds to less than one event per week. Similarly, the CDMS collaboration, after various cuts, had no events for 121 kg- days of germanium data [2]. EDELWEISS discusses four events on the basis of 322 kg-days [3]. And most recently XENON100, with 48 kg for 100 days, reports only three events [4].

With such low event rates even rare sources of background must be considered. In particular the most worrisome background in underground experiments searching for dark matter through nuclear recoils are neutron-induced nuclear recoils. Neutrons scatter off nuclei and the recoils could closely resemble the

sought-for signal. Furthermore as we shall explain below, fast neutrons, which are difficult to shield, will give elastic nuclear recoils with the same general energies as expected in the dark matter searches. Although estimates of neutrons from standard sources such as incoming muons or radioactivity in the surrounding rock ( see the above references and for example [5]) give neutron fluxes which are quite small, the very good level of background suppression being reached implies that even very rare processes or perhaps unexpected contaminations must be considered. While there are elaborate Monte-Carlo program sets for carrying out standard background calculations, it must be said that they are not very transparent, and do not offer much insight into the nature of the effects under consideration. Hence it may be useful to have a discussion based on a simple physical picture, even if this is only semi-quantitatively accurate.

Estimates generally lead to neutron spectra dominated by low energies. Such fluxes can be strongly reduced by neutron shields. However such shields are typically hydrocarbon materials, which while very effective at low energy, become rapidly ineffective at higher energies. Thus the PDG tables [6] give the rather large value of 88 cm for the nuclear interaction length in polyethylene, and for example in a Monte Carlo study with a lead-hydrocarbon shield [7] the authors remark that this “provides effective shielding only for neutrons below 10 MeV”.

Thus there is a possibly significant question of “fast” or “higher energy ” neutrons. It may therefore be useful if we present some simple remarks about the effects to be anticipated from them in dark matter detectors and perhaps other low background setups. We can do this in a simple and general way terms using the ‘black disc’ model which, for fast neutrons, approximately applies for all nuclei. While this can only be regarded as a semi-quantative approximation, it offers a simple and general overview of the situation. Unfortunately, even such an rough discussion is not possible for slow neutrons.

We should stress that we use the terms “fast” or “high energy ” in the sense of neutron physics, that is to mean neutrons of a few MeV and above, and of course not in the sense of high energy particle physics.

## 2 “High energy ” neutrons and the “Black disc”

Above a few MeV neutron-nucleus scattering begins to resemble the “black disc” limit. While at lower energies the behavior of the cross section is dominated by resonances and shows a complicated structure which varies from nucleus to nucleus, at the higher energies a smooth behavior with respect to neutron energy sets in, one which can approximately be described by the ‘black disc’ model. This model in its simplest form only involves one parameter, namely  $R$ , the radius of the nucleus. This permits a simple description, applicable to all target nuclei, adequate for our purposes of semi-quantative background estimates.

In Fig 1, Fig 2, and Fig 3 [8] we show as examples the total experimental cross section for the light, medium, and heavy weight nuclei oxygen, calcium, and tungsten. One notes that just above the resonance region the total cross

section becomes smooth and tends to be above the black disc value, (indicated by the grey line using  $R$  as given by Eq 3) and then slowly descend through it as the energy increases.

In the elastic scattering there is a forward diffraction peak at low momentum transfer  $\Delta$ . This elastic scattering is concentrated in a region characterized by

$$\Delta R \sim 1, \quad (1)$$

where  $R$  is the radius of the nucleus. At the same time there is an inelastic cross section of about the same magnitude. In the perfect "black disc" limit the elastic and inelastic scattering cross sections are equal and of magnitude  $\pi R^2$ . As we shall discuss below, this inelastic scattering could in principle be used as a signal for the presence of the fast neutrons.

## 2.1 Validity of the simple 'Black disc' model

The principle features of the simple 'Black disc' model result from the assumption that the nucleus is totally absorbing for neutrons up to impact parameter  $R$ , beyond which there is no scattering or absorption. This simple assumption has the following consequences: 1) the total cross section is made up of equal elastic and inelastic parts, adding up to  $\sigma_{tot} = 2 \times \pi R^2$ , 2) these features are energy independent 3) the form of the elastic scattering versus angle or momentum transfer is given by a simple Bessel function expression, (see Eq 7) where in particular the form versus momentum transfer is independent of the neutron energy.

As one sees in the examples of Fig 1, Fig 2, and Fig 3 the energy where the smooth 'optical model' behavior sets in varies with the target nucleus. For oxygen the energy must be above about 10 MeV, for calcium above 4 or 5 MeV, while for tungsten 1 MeV suffices. One also notes from the plots that with our value for  $R$  the total cross section is somewhat underestimated, particularly at the lower energies.

There is a large literature concerning more sophisticated treatments of the optical model [9]. There the simple 'black' assumption is replaced by a partial and energy dependent transparency, the 'hard edge' is softened, modifying the simple radius assumption, and resonance effects and variation with nucleus may be included. While all these points are interesting and important they will not significantly alter our general conclusions, particularly our main point, as to be discussed in section 3 that the recoil energies implied by the optical model and characterized by our parameter  $E_R^o$  are in the region anticipated for dark matter searches.

## 3 Recoil energies

It is interesting that if we put Eq 1 in the formula for the recoil energy  $E_R$

$$E_R = \frac{\Delta^2}{2M_A} \sim \frac{1}{R^2 2M_A}, \quad (2)$$

<i>Element</i>	<i>A</i>	<i>R</i> (fermi)	$E_R^o(keV)$
O	16	3.5	98
F	19	3.7	74
Na	23	4.0	54
Si	28	4.2	39
Ar	40	4.8	21
Ca	40	4.8	21
Ge	74	5.9	7.7
I	127	7.0	3.1
Xe	132	7.1	2.9
W	184	7.9	1.7

Table 1: The quantities  $R$ ,  $E_R^o$  for various nuclei with mass number  $A$ .

one gets recoils in the region of interest for many dark matter searches.  $M$  is the mass of the nucleus,  $M \approx A \times 1 \text{ GeV}$  and for  $R$  we use

$$R = A^{1/3} \times 1.4 \text{ f}. \quad (3)$$

Calling this typical recoil energy  $E_R^o$  one obtains ( $\hbar, c = 1$  units)

$$E_R^o = \frac{1}{R^2 2M_A} = \frac{10}{A^{5/3}} \text{ MeV}. \quad (4)$$

This is the typical energy characterizing the elastic scattering of the “high energy neutron”. In Table 1 we show the value of  $E_R^o$  for some nuclei. The numbers run through the values often considered in dark matter searches.

The differential cross section is a function of the dimensionless parameter  $x = \Delta R$ , and with these definitions we can write

$$(\Delta R)^2 = \frac{E_R}{E_R^o}. \quad (5)$$

The introduction of the dimensionless variable  $x$ , where  $x^2 = \frac{E_R}{E_R^o}$  allows us to describe, within the limitations of the model, all nuclei in a universal way in terms of their  $E_R^o$ .

A second interesting point is that in the model this typical recoil energy for the “high energy” neutrons depends on the nucleus *but not* on the energy of the neutron. This implies that the whole of the “high energy” neutron spectrum contributes in approximately the same way to the recoils. This statement is only strictly true insofar as the black disc model is exactly valid, but it will be seen that the major features of the model hold approximately over a wide range of energy.

Finally we note the relation between scattering angle and recoil energy

$$E_R = \frac{\Delta^2}{2M_A} = \frac{(P_{neutron})^2}{M_A} (1 - \cos\theta) = 2E_{neutron} \frac{m_{neutron}}{M_A} (1 - \cos\theta), \quad (6)$$

with  $M_A$  the mass of the nucleus. This simple expression will also be true, replacing “neutron” with “WIMP”, for the elastic WIMP scattering when the mass of the WIMP is small compared to that of the nucleus.

In the ‘black disc’ limit the energy spectrum of the recoils is quite simple. The diffraction peak leads to an  $E_R$  spectrum falling off with energy, but at least for the lighter nuclei not so steeply as from WIMP scattering. The elastic scattering is given by a Bessel function expression [10],  $\frac{d\sigma}{d\Delta^2} \sim \left(\frac{J_1(\Delta R)}{\Delta R}\right)^2$  and since  $E_R = \Delta^2/2M_A$ , one expects

$$\frac{d\sigma}{dE_R} = 2M_A \frac{d\sigma}{d\Delta^2} \sim \left(\frac{J_1(\Delta R)}{\Delta R}\right)^2, \quad (7)$$

Now since  $(\Delta R)^2 = E_R/E_R^o$  we can introduce the convenient variable  $x$ , where  $x^2 = E_R/E_R^o$  and write

$$\frac{d\sigma}{dE_R} \sim \frac{d\sigma}{dx^2} \sim \left(\frac{2J_1(x)}{x}\right)^2 \quad x^2 = E_R/E_R^o, \quad (8)$$

We include the ‘2’ in the Bessel function expression so it is normalized to one at  $x = 0$ . For small  $x$ ,  $x \lesssim 1$ , the expression then behaves as

$$\left(\frac{2J_1(x)}{x}\right)^2 = 1 - \frac{1}{4}x^2 + \dots = 1 - \frac{1}{4}\frac{E_R}{E_R^o} + \dots \quad (9)$$

For small  $\frac{E_R}{E_R^o}$  this implies a relatively flat energy spectrum. Thus for oxygen, between  $E_R = 10$  and 40 keV,  $\frac{d\sigma}{dE_R}$  goes down only by about 8%. Note again that it is not necessary to know the neutron energy spectrum to reach this conclusion, as long as we have “high energy” neutrons. This is of course quite different from the rapid fall-off of WIMP scattering, even on light nuclei. The difference originates in the fact that while for fast neutrons the recoil spectrum is determined by the radius of the nucleus, for WIMPs it is largely determined by the velocity spectrum of the incoming particles, which falls steeply above a certain velocity.

Hence it appears that a characteristic sign of the fast neutron background would be a *rather flat recoil energy spectrum for the light elements*.

For  $\frac{E_R}{E_R^o}$  larger than one the black disc elastic cross section falls rapidly. For convenience we show a few values in Table 2. One sees that most of the elastic cross section is contained within  $x^2 = \frac{E_R}{E_R^o} \lesssim a few$

In Fig 4 we show some data on the angular distribution for oxygen. The optical model shape is quite clear, with a well-defined low momentum transfer peak and diffractive oscillations, although at this energy the effective value for  $R$  is slightly smaller than the one we are using. To compare with our variable  $\Delta$  we have added an axis showing the momentum transfer for 20 MeV neutrons.

$x$	$E_R/E_R^o$	$(2J_1(x)/x)^2$
0	0	1
1.0	1.0	0.77
1.6	2.6	0.51
2.0	4.0	0.33
2.6	7.0	0.13
3.0	9.0	0.051
3.6	13	0.0028

Table 2: Some values for the Bessel function expression for the elastic cross section in the ‘black disc’ model, normalized to 1 at zero recoil energy.

## 4 Patterns of A Behavior

Given a certain neutron flux in a detector with different nuclei, the rate will vary in a characteristic way with increasing mass number  $A$ . In the black disc limit the total elastic cross section, that is, integrated over all  $E_R$ , is  $\pi R^2$ , and so one has

$$\sigma \sim R^2 \sim A^{2/3} \quad \text{all } E_R. \quad (10)$$

However, most detectors will have a lower energy threshold for detection and so will miss part of the diffraction peak. So it is perhaps also useful to consider the behavior of  $\frac{d\sigma}{dE_R}$  at some fixed  $E_R$ , where one has

$$\frac{d\sigma}{dE_R} = \frac{\pi}{2} M R^4 \left( \frac{2J_1(x)}{x} \right)^2 \sim A^{7/3} \left( \frac{J_1(x)}{x} \right)^2 \quad x = \left( \frac{E_R}{E_R^o} \right)^{1/2} \quad (11)$$

This formula follows from Eq7 and the use of the optical theorem point for forward scattering with a purely imaginary amplitude, as one has for the ‘black disc’:

$$\left. \frac{d\sigma}{d\Delta^2} \right|_{\Delta=0} = \frac{1}{16\pi} \sigma_{total}^2. \quad (12)$$

In terms of the  $x$  variable alone, these relations can also be given the simple form

$$\frac{d\sigma}{dx^2} = \frac{1}{4} \pi R^2 \left( \frac{2J_1(x)}{x} \right)^2 \quad (13)$$

At fixed  $E_R$ , the increase of  $\sim A^{7/3}$  in Eq 11 will be slowed by the decreasing  $E_R^o$ , which causes  $x$  to move out in the Bessel function expression. Since in the ‘black disc’ limit the recoil spectrum is independent of the incident neutron energy and the incoming flux is the same for all nuclei in a given experimental setup, we may take the rate as approximately proportional to Eq 11. In Table 3 we show, for  $E_R = 20$  and  $30 \text{ keV}$ , how Eq 11 and thus the rate varies with  $A$  for some elements.

<i>Element</i>	<i>A</i>	neutron $E_R = 20 \text{ keV}$	neutron $E_R = 30 \text{ keV}$	WIMP M=10 GeV	WIMP M=50 GeV
O	16	1	1	1	1
F	19	1.5	1.5	1.3	1.8
Na	23	2.2	2.2	1.6	3.3
Si	28	3.4	3.3	1.8	6.7
Ar	40	7.0	6.4	1.1	19
Ca	40	7.0	6.4	1.1	19
Ge	74	19	13	$\sim 0$	93
I	127	20	5.1	$\sim 0$	200
Xe	132	18	3.9	$\sim 0$	240
W	184	2.6	1.6	$\sim 0$	230

Table 3: Variation of the differential scattering rate per unit energy over various nuclei in the ‘black disc’ limit, Eq 11, at  $E_R = 20 \text{ keV}$  and  $E_R = 30 \text{ keV}$ . For comparison the same rate for a coherently scattering WIMP at  $E_R = 20 \text{ keV}$  for masses 10 and 50 GeV is also shown. One notes different patterns of A behavior for neutrons and WIMPs. All values are per nucleus and normalized to that for oxygen.

The interplay of the factors in Eq 11 leads to a maximum of the rate at fixed  $E_R$  towards the middle of the periodic table. An interesting point is that while for lower energy neutrons one expects, on essentially kinematic grounds, the scattering to be predominantly on light nuclei such as oxygen, for “high energy” neutrons, where the incident energy is high relative to the recoil energy, this is no longer true.

All values in the Table are normalized to that on oxygen. The entries refer to the rates per nucleus, so for a given detector account should be taken of the relative abundance of the nucleus in the target material.

For comparison we also show the rates for 10 and 50 GeV WIMPs, with coherent  $\sim A^2$  interactions, for  $E_R = 20 \text{ keV}$ . For WIMPs not having the simple coherent interactions, it is not possible to make a simple statement as to the A behavior since the variation from nucleus to nucleus will depend on the quantum number constitution of the WIMP. Indeed it might be possible to unravel these quantum numbers by studying the A behavior [13].

Eq 13 may be used to estimate the event rate in an acceptance window for a given setup and an assumed neutron flux. Bessel function relations can be used to estimate what fraction of the elastic cross section  $\pi R^2$  is in a certain interval. Thus, for example, assume a  $\text{CaWO}_4$  detector accepting events on the oxygen between 10 keV and 40 keV. From Table 1 this corresponds to  $x^2 = E_R/E_r^o$  between 0.11 and 0.42. On the other hand the total elastic cross section according to Eq 13 is proportional to  $\int_0^\infty (2J_1(x)/x)^2 dx^2$ . Now there is the “orthonormality relation” [12]  $\int_0^\infty (2J_1(x)/x)^2 dx^2 = 4$ , where we have normalized as in Eq 9. According to Eq 9 one may take  $(2J_1(x)/x)^2$  as approximately equal to one in

this interval. Thus we have a fraction of the elastic cross section  $0.31/4 = 0.078$  in the interval 10 keV to 40 keV. Since  $\pi R^2 = 0.39 b$ , there is thus a cross section of 0.030 b in the interval.

For a flux of say  $1 \times 10^{-7}/cm^2s$  for the “high energy ” neutrons, this leads to a rate of  $2 \times 10^{-3}/kg\text{ day}$  for  $\text{CaWO}_4$ . Some information on underground neutron fluxes may be found in Reference [14]. Naturally given the neutron flux at a given location, the neutrons must be further be propagated through the shielding and materials of the setup in question, and there is the further question of neutrons originating in these materials.

## 5 Inelastic Processes

An important difference between neutrons and WIMPs is of course that neutrons are strongly interacting and WIMPs weakly interacting. This implies that, depending on the material and geometry of the detector, multiple scattering may serve as a signal for neutrons.

A further significant difference between the MeV neutrons we discuss here and the assumed galactic WIMPs is the low energy of the WIMP. The WIMP is expected to have a small velocity, typical of objects in the Milky Way,  $v \sim 2 \times 10^{-3}$  (c=1 units). Then even, say, a mass 50 GeV WIMP only has an energy  $\frac{1}{2}mv^2 \sim 100\text{ keV}$ , and a ‘light WIMP’ with  $M=10\text{ GeV}$  only  $\sim 20\text{ keV}$ . Such energies are generally not enough to induce significant inelastic processes on most nuclei.

On the other hand, neutrons with MeVs and above typically have many inelastic reactions and the inelastic reactions are expected at close to the same rate as the elastic scattering. In the ‘black disc’ limit’ one has  $\sigma_{elastic} \approx \sigma_{inelastic} \approx \frac{1}{2}\sigma_{total}$ . Fig 5 [8] for the elastic scattering on oxygen, when compared with Fig 1, shows this is approximately true.

In experiments with two-signal readout designed to distinguish electromagnetic backgrounds from nuclear events, such as CRESST, Edelweiss, CDMS, Xenon... these inelastic events will typically appear as a high energy signal in the nuclear event band. Depending on the nuclei in question, the most common inelastic channels will be nuclear excitation, leading to a  $\gamma$  ray, or nuclear breakup. In breakup common nuclear fragments are further neutrons and  $\alpha$  particles, while for nuclear excitation followed by  $\gamma$  rays electromagnetic energy is present. Therefore inelastic events will generally have a relatively high energy deposit, compared to the elastic recoils either from WIMPs or neutrons. Depending on the nuclei present in the detector, these inelastic events could also appear on nuclei other than those exhibiting the small recoils. Thus associated with a fast neutron flux there should be a characteristic set of events with quite different properties from the small elastic scattering recoils. But these should be absent for WIMPs.



## 6 Conclusions

We have pointed out that the difficult-to-shield fast neutrons will give elastic nuclear recoils of the same type as sought for in dark matter searches. We explain, however, that it should be possible to use evidence internal to the detector in establishing the presence of a neutron background and so not to be only dependent upon standard assumptions and Monte Carlo estimates. We have suggested some methods of doing this, including the pattern of recoil energies, the rates on different nuclei through the periodic table (see section 4) and the presence of inelastic nuclear processes at about the same level as elastic nuclear recoils (see section 5). In this respect it is thus desirable to have detector systems able to distinguish more than one type of recoiling nucleus [15].

For light to medium weight nuclei the neutron induced events should show a rather flat spectrum with respect to recoil energy  $E_R$ . At a fixed recoil energy, the variation of the rate with respect to changing the nucleus is different for neutrons and WIMPs as explained in connection with Table 3.

Neutrons can of course have multiple interactions, and fast neutrons as opposed to WIMPs can induce inelastic reactions, and at a relatively high rate.

The great difficulty, as with all aspects of dark matter searches, is that at present only a handful of events is available and that a detailed investigation of these points requires extensive data, not easy to come by in very low rate experiments.

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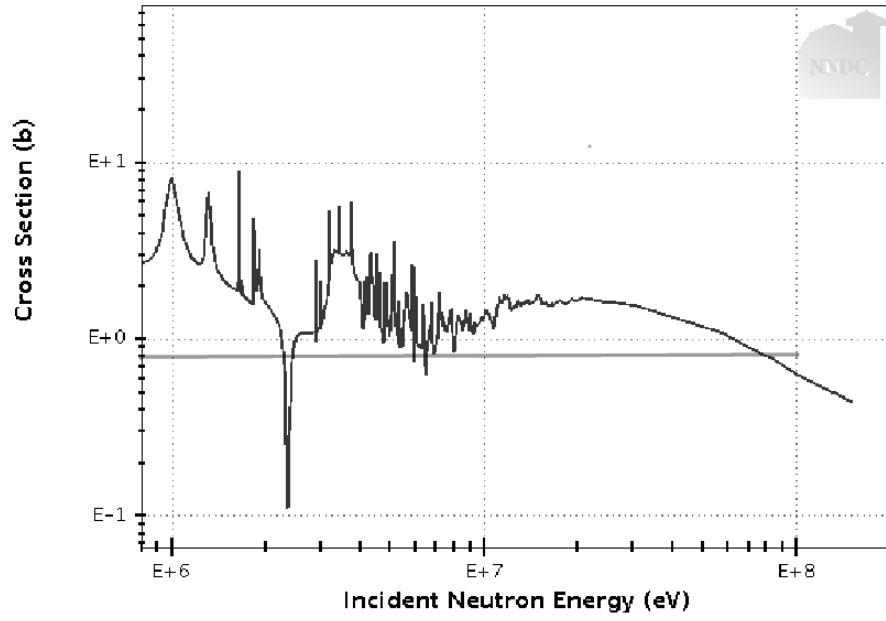


Figure 1: The total cross section for neutrons on  $^{16}\text{O}$ , from the data compilations of the neutron data center NNDC at Brookhaven National Laboratory. Above the resonance region the cross section behaves smoothly. The ‘black disc’ estimate (grey line), using  $R = A^{1/3}1.4\text{ f}$ , is  $2 \times \pi R^2 = 0.78\text{ b}$ .

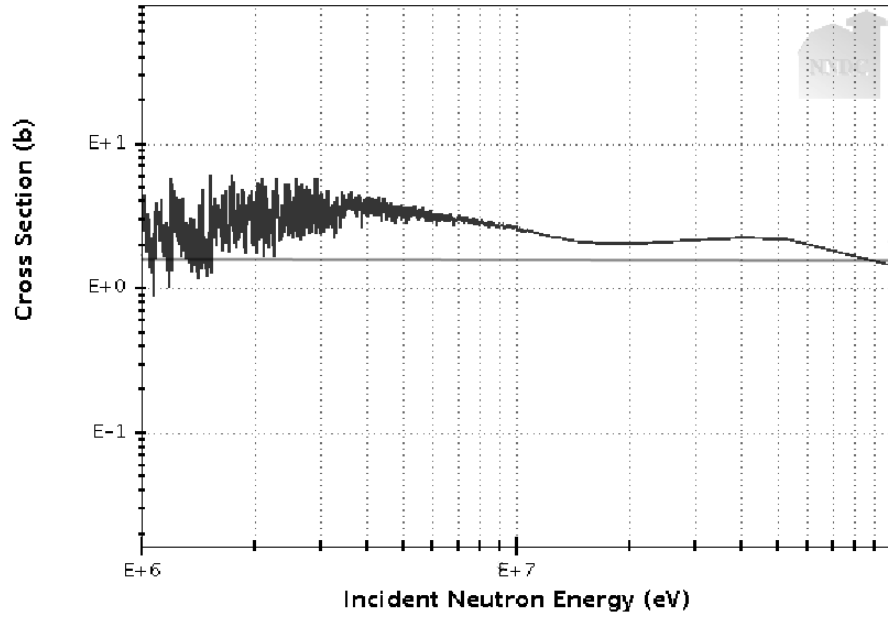


Figure 2: The total cross section for neutrons on  $^{40}\text{Ca}$  from 1 to 100 MeV, from the NNDC. The black disc value with  $R = A^{1/3} 1.4f$  is 1.4 b

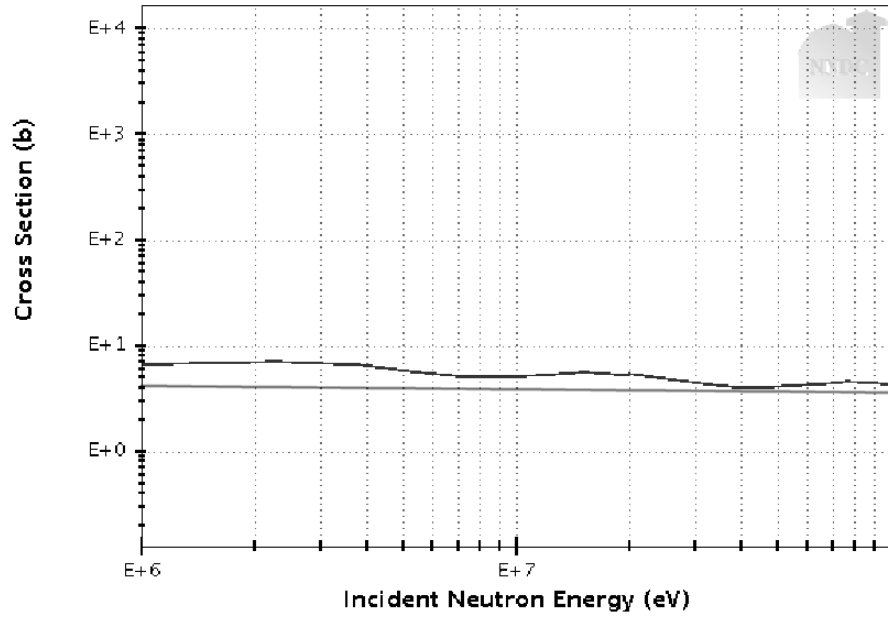


Figure 3: The total cross section for neutrons on  $^{184}\text{W}$  from 1 to 100 MeV, from the NNDC. The black disc value with  $R = A^{1/3} 1.4f$  is 4.0 b

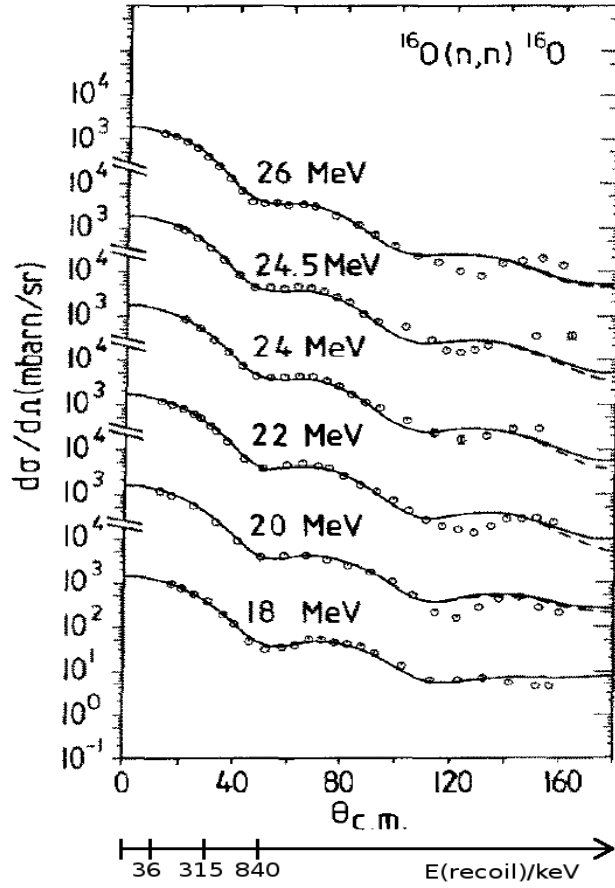


Figure 4: The angular distribution for elastic neutron-oxygen scattering for several energies from ref [11]. The added horizontal axis shows, for the 20 MeV case,  $E_R$  for scattering angles 10, 30 and 50 degrees. In the ‘black disc’ limit the curves should be the same for all energies, when plotted against  $E_R$  instead of angle.

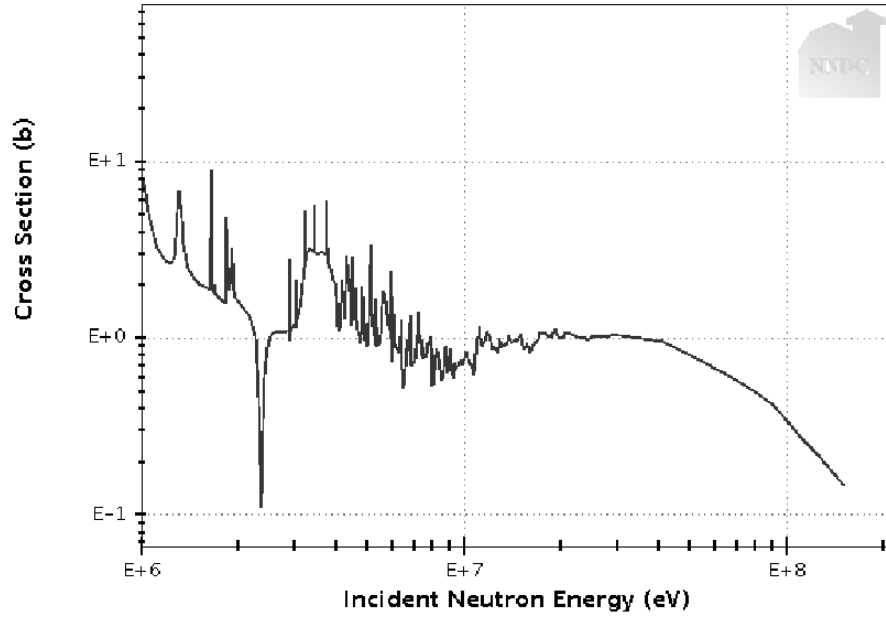


Figure 5: The total cross section for elastic scattering on  $^{16}\text{O}$ , from the NNDC. As compared to the total cross section in Fig 1, this elastic cross section includes only processes where the nucleus undergoes no breakup or excitation. One notes that the simple optical model relation  $\sigma_{elastic} \approx \frac{1}{2}\sigma_{total}$  is approximately correct.